

Absorption by Branes and Schwinger Terms in the World Volume Theory

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Abstract

We study how coincident Dirichlet 3-branes absorb incident gravitons polarized along their world volume. We show that the absorption cross-section is determined by the central term in the correlator of two stress-energy tensors. The existence of a non-renormalization theorem for this central charge in four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theories shows that the leading term at low energies in the absorption cross-section is not renormalized. This guarantees that the agreement of the cross-section with semiclassical supergravity, found in earlier work, survives all loop corrections. The connection between absorption of gravitons polarized along the brane and Schwinger terms in the stress-energy correlators of the world volume theory holds in general. We explore this connection to deduce some properties of the stress-energy tensor OPE's for 2-branes and 5-branes in 11 dimensions, as well as for 5-branes in 10 dimensions.

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1 Introduction

Extremal black holes with non-vanishing horizon area may be embedded into string theory or M-theory using intersecting p -branes [1, 2, 3, 4, 5, 6, 7, 8]. These configurations are useful for a microscopic interpretation of the Bekenstein-Hawking entropy. The dependence of the entropy on the charges, the non-extremality parameter, and the angular momentum suggests a connection with $1 + 1$ dimensional conformal field theory [2, 3, 9, 4, 5, 7]. This “effective string” is essentially the intersection of the p -branes. Calculations of emission and absorption rates [3, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25] provide further tests of the “effective string” models of $D = 5$ black holes with three charges and of $D = 4$ black holes with four charges. For minimally coupled scalars the functional dependence of the greybody factors on the frequency agrees exactly with semiclassical gravity, providing a highly non-trivial verification of the effective string idea [13, 15]. Similar successes have been achieved for certain non-minimally coupled scalars, which were shown to couple to higher dimension operators on the effective string. For instance, the “fixed” scalars [26] were shown to couple to operators of dimension $(2, 2)$ [16] while the “intermediate” scalars [21] – to operators of dimension $(2, 1)$ and $(1, 2)$. Unfortunately, there is little understanding of the “effective string” from first principles, and some of the more sensitive tests reveal this deficiency. For instance, semiclassical gravity calculations of the “fixed” scalar absorption rates for general black hole charges reveal a gap in our understanding of higher dimension operators [20]. A similar problem occurs when one attempts a detailed effective string interpretation of the higher partial waves of a minimally coupled scalar [22, 23]. Even the s-wave absorption by black holes with general charges is complex enough that it is not reproduced by the simplest effective string model [17, 25]. These difficulties by no means invalidate the general qualitative picture, but they do pose some interesting challenges. In order to gain insight into the relation between gravity and D-branes it is useful to study, in addition to the intersecting branes, the simpler configurations which involve parallel branes only.

A microscopic interpretation of the entropy of near-extremal p -branes was first studied in [27, 28]. It was found that the scaling of the Bekenstein-Hawking entropy with the temperature agrees with that for a massless gas in p dimensions only for the “non-dilatonic p -branes”: namely, the self-dual 3-brane of the type IIB theory, and the 2- and 5-branes of M-theory. Their further study was undertaken in [29, 30] where low-energy absorption cross-sections for certain incident massless particles were compared between semiclassical supergravity and string or M-theory. For the 3-branes, exact agreement was found for the leading low-energy behavior of the absorption cross-sections for dilatons [29], as well as for R-R scalars and for gravitons polarized along the brane [30]. The string-theoretic description of macroscopic 3-branes can be given in terms of many coincident D3-branes [31, 32]. Indeed, N parallel D3-branes are known to be described by a $U(N)$ gauge theory in $3+1$ dimensions with $\mathcal{N} = 4$ supersymmetry [33]. This theory has a number of remarkable properties, including exact S-duality, and we will be able to draw on a known non-renormalization theorem in explaining the absorption by the 3-branes.

From the point of view of supergravity, the 3-brane is also special because its extremal geometry,

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-1/2} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \left(1 + \frac{R^4}{r^4}\right)^{1/2} (dr^2 + r^2 d\Omega_5^2) , \quad (1)$$

is non-singular [34], while the dilaton background is constant. Instead of a singularity at $r = 0$ we find an infinitely long throat whose radius is determined by the charge (the vanishing of the horizon area is due to the longitudinal contraction). Thus, for a large number N of coincident branes, the curvature may be made arbitrarily small in Planck units. For instance, for N D3-branes, the curvature is bounded by a quantity of order

$$\frac{1}{\sqrt{N\kappa}} \sim \frac{1}{\alpha' \sqrt{N g_{\text{str}}}} . \quad (2)$$

In order to suppress the string scale corrections to the classical metric, we need to take the limit $N g_{\text{str}} \rightarrow \infty$.

The tension of a D3-brane depends on g_{str} and α' only through the ten-dimensional gravitational constant $\kappa = 8\pi^{7/2} g \alpha'^2$:

$$T_{(3)} = \frac{\sqrt{\pi}}{\kappa} . \quad (3)$$

This suggests that we can compare the expansions of various quantities in powers of κ between the microscopic and the semiclassical descriptions. Indeed, the dominant term in the absorption cross-section at low energy is [29, 30]

$$\sigma = \frac{\pi^4}{8} \omega^3 R^8 = \frac{\kappa^2 \omega^3 N^2}{32\pi} , \quad (4)$$

which agrees between semiclassical supergravity and string theory.

It is important to examine the structure of higher power in g_{str} corrections to the cross-section [35]. In semiclassical supergravity the only quantity present is κ , and corrections to (4) can only be of the form

$$a_1 \kappa^3 \omega^7 + a_2 \kappa^4 \omega^{11} + \dots \quad (5)$$

However, in string theory we could in principle find corrections even to the leading term $\sim \omega^3$, so that

$$\sigma_{\text{string}} = \frac{\kappa^2 \omega^3 N^2}{32\pi} \left(1 + b_1 g_{\text{str}} N + b_2 (g_{\text{str}} N)^2 + \dots\right) + \mathcal{O}(\kappa^3 \omega^7) . \quad (6)$$

Presence of such corrections would spell a manifest disagreement with supergravity because, as we have explained, the comparison has to be carried out in the limit $N g_{\text{str}} \rightarrow \infty$. The purpose of this paper is to show that, in fact, $b_i = 0$ due to a non-renormalization theorem in $D = 4$ $\mathcal{N} = 4$ SYM theory.

In section 2 we present a detailed argument for the absence of such corrections in the absorption cross-section of gravitons polarized along the brane. These gravitons couple to the stress-energy tensor on the world volume and we will show that the absorption cross-section is, up to normalization, the central term in the two-point function of the stress-energy tensor. The fact that $b_i = 0$ follows from the fact that the one-loop calculation of the central charge is exact in $D = 4$ $\mathcal{N} = 4$ SYM theory.

The connection between absorption of gravitons polarized along the brane and Schwinger terms in the stress-energy correlators of the world volume theory is a general phenomenon that holds for all branes. In section 3 we explore this connection to deduce some properties of the stress-energy tensor OPE's for multiple 2-branes and 5-branes of M-theory, as well as for multiple 5-branes of string theory.

2 D-brane approach to absorption

It was probably Callan who first realized that, in terms of D-brane models, absorption cross-sections correspond up to a simple overall factor to discontinuities of two point functions of certain operators on the D-brane world volume [38]. This realization was exploited in [14, 24]. Consider massless scalar particles in ten dimensions normally incident upon D3-branes. If the coupling to the brane is given by

$$S_{\text{int}} = \int d^4x \phi(x, 0) \mathcal{O}(x) , \quad (7)$$

where $\phi(x, 0)$ is a canonically normalized scalar field evaluated on the brane, and \mathcal{O} is a local operator on the brane, then the precise correspondence is

$$\sigma = \frac{1}{2i\omega} \text{Disc } \Pi(p) \Big|_{\substack{p^0=\omega \\ \vec{p}=0}} . \quad (8)$$

Here ω is the energy of the incident particle, and

$$\Pi(p) = \int d^4x e^{ip \cdot x} \langle \mathcal{O}(x) \mathcal{O}(0) \rangle . \quad (9)$$

When \mathcal{O} is a scalar in the world volume theory, $\Pi(p)$ depends only on $s = p^2$, and $\text{Disc } \Pi(p)$ is computed as the difference of Π evaluated for $s = \omega^2 + i\varepsilon$ and $s = \omega^2 - i\varepsilon$. In the case of the graviton, we shall see that $\Pi(p)$ is a polynomial in p times a function of s , so the evaluation of $\text{Disc } \Pi(p)$ is equally straightforward.

The validity of (8) depends on ϕ being a canonically normalized field. In form it is almost identical to the standard expression for the decay rate of an unstable particle of mass ω . The dimensions are different, however: σ is the cross-section of the 3-brane per unit world volume, and so has dimensions of (length)⁵. Similar formulas can be worked out for branes of other dimensions and for near-extremal branes, although away from extremality (9) would become a thermal Green's function (see [14, 24]).

Now let us work through the example of the graviton. The 3-brane world volume theory is $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory, where N is the number of parallel 3-branes [33]. Thus, the massless fields on the world volume are the gauge field, six scalars, and four Majorana fermions, all in the adjoint representation of $U(N)$. To lowest order in κ , and ignoring the couplings to the bulk fields, the world volume-action is ($I = 1, \dots, 4$; $i = 4, \dots, 9$)

$$S_3 = \int d^4x \operatorname{tr} \left[-\frac{1}{4} F_{\alpha\beta}^2 + \frac{i}{2} \bar{\psi}^I \gamma^\alpha \partial_\alpha \psi_I + \frac{1}{2} (\partial_\alpha X^i)^2 + \text{interactions} \right] . \quad (10)$$

The interactions referred to here are the standard renormalizable ones of $\mathcal{N} = 4$ super-Yang-Mills (see for example [40] for the complete flat-space action). In equation (3.4) of [30], a factor of the 3-brane tension $T_{(3)}$ appeared in front of the action. It is convenient to work with canonically normalized fields, and so, relative to the conventions of [30], we have absorbed a factor of $\sqrt{T_{(3)}}$ into A_μ , ψ^I , and X^i . Another difference between [30] and the present paper is that we work here with a mostly minus metric and the spinor conventions of [40].

The full (and as yet unknown) action for multiple 3-branes is non-polynomial, and (10) includes only the dimension 4 terms. The higher dimension terms will appear with powers of $1/T_{(3)}$, which is to say positive powers of κ . They could give rise to corrections of the form (5), but not (6).

Despite our ignorance of the full action for multiple coincident 3-branes, one can be fairly confident in asserting that the external gravitons polarized parallel to the brane couple via

$$S_{\text{int}} = \int d^4x \frac{1}{2} h^{\alpha\beta} T_{\alpha\beta} , \quad (11)$$

where $h_{\alpha\beta} = g_{\alpha\beta} - \eta_{\alpha\beta}$ is the perturbation in the metric, and $T_{\alpha\beta}$ is the stress-energy tensor:

$$\begin{aligned} T_{\alpha\beta} = \operatorname{tr} [& -F_\alpha{}^\gamma F_{\beta\gamma} + \frac{1}{4} \eta_{\alpha\beta} F_{\gamma\delta}^2 + \frac{i}{2} \bar{\psi}^I \gamma_{(\alpha} \partial_{\beta)} \psi_I \\ & + \frac{2}{3} \partial_\alpha X^i \partial_\beta X^i - \frac{1}{6} \eta_{\alpha\beta} (\partial_\gamma X^i)^2 - \frac{1}{3} X^i \partial_\alpha \partial_\beta X^i + \frac{1}{12} \eta_{\alpha\beta} X^i \square X^i \\ & + \text{interactions}] . \end{aligned} \quad (12)$$

This “new improved” form of the stress-energy tensor [39] is chosen so that $\partial_\mu T^{\mu\nu} = 0$ and $T^\mu{}_\mu = 0$ on shell. The scalar terms differ from the canonical form

$$T_{\alpha\beta}^{(\text{can})} = \operatorname{tr} \left[\partial_\alpha X^i \partial_\beta X^i - \frac{1}{2} \eta_{\alpha\beta} (\partial_\gamma X^i)^2 + \dots \right] . \quad (13)$$

The difference arises from adding a term $-\frac{1}{12} \sqrt{-g} R \operatorname{tr} X^2$ to the lagrangian so that the scalars are conformally coupled.

Choosing the scalars to be minimally or conformally coupled does not affect the one-loop result for the cross-section. But it is the traceless form of $T_{\alpha\beta}$ presented in (12) which has a non-renormalized two-point function. The reason is that the conformal $T_{\alpha\beta}$ is in the same supersymmetry multiplet as the supercurrents and the $SU(4)$ R-currents. $\mathcal{N} = 4$ super-Yang-Mills theory is finite to all orders and anomaly free in flat space. The Adler-Bardeen

theorem guarantees that any anomalies of the $SU(4)$ R-currents can be computed exactly at one loop. Because $\partial_\alpha R^\alpha$ and T_α^α are in the same supermultiplet (the so-called “multiplet of anomalies”), the one-loop result for the trace anomaly must also be exact.¹ Thus the trace anomaly in a curved background is the same as in the free theory:

$$\langle T_\mu^\mu \rangle = -\frac{1}{16\pi^2} \left(cF - \frac{2c}{3} \square R - bG \right), \quad (14)$$

where $F = C_{\alpha\beta\gamma\delta}^2$ is the square of the Weyl tensor and $G = R_{\alpha\beta\gamma\delta}^2 - 4R_{\alpha\beta}^2 + R^2$ is the topological Euler density. Thus the second and third terms in (14) are total derivatives. The coefficients c and b are given by [45]

$$\begin{aligned} c &= \frac{12N_1 + 3N_{1/2} + N_0}{120} = \frac{N^2}{4} \\ b &= \frac{124N_1 + 11N_{1/2} + 2N_0}{720} = \frac{N^2}{4}. \end{aligned} \quad (15)$$

$N_1 = N^2$, $N_{1/2} = 4N^2$, and $N_0 = 6N^2$ are the numbers of spin-one, Majorana spin-half, and real spin-zero fields in the super-Yang-Mills theory. Note that spin-one, spin-half, and spin-zero particles make contributions to c in the ratio 2 : 2 : 1. The different spins contribute to the cross-section in precisely the same ratio, as was demonstrated in [30].

It remains to make the connection between $\langle T_\alpha^\alpha \rangle$ and $\langle T_{\alpha\beta}(x)T_{\gamma\delta}(0) \rangle$. Suppressing numerical factors and Lorentz structure, the OPE of $T_{\alpha\beta}$ with $T_{\gamma\delta}$ is [41]

$$T(x)T(0) = \frac{c}{x^8} + \dots + \frac{T(0)}{x^4} + \dots \quad (16)$$

We have omitted terms involving the Konishi current as well as terms less singular than $1/x^4$, and we have anticipated the conclusion that the coefficient on the Schwinger term is precisely the central charge c appearing in (14). A clean argument to this effect is presented in [42] and summarized briefly below. The Schwinger term is nothing but the two-point function: with all factors and indices written out explicitly² [43],

$$\langle T_{\alpha\beta}(x)T_{\gamma\delta}(0) \rangle = \frac{c}{48\pi^4} X_{\alpha\beta\gamma\delta} \left(\frac{1}{x^4} \right) \quad (17)$$

where

$$\begin{aligned} X_{\alpha\beta\gamma\delta} &= 2\square^2 \eta_{\alpha\beta}\eta_{\gamma\delta} - 3\square^2 (\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}) - 4\partial_\alpha\partial_\beta\partial_\gamma\partial_\delta \\ &\quad - 2\square(\partial_\alpha\partial_\beta\eta_{\gamma\delta} + \partial_\alpha\partial_\gamma\eta_{\beta\delta} + \partial_\alpha\partial_\delta\eta_{\beta\gamma} + \partial_\beta\partial_\gamma\eta_{\alpha\delta} + \partial_\beta\partial_\delta\eta_{\alpha\gamma} + \partial_\gamma\partial_\delta\eta_{\alpha\beta}). \end{aligned} \quad (18)$$

The argument of [42] starts by obtaining an expression for the flat space three point function $\langle T_\alpha^\alpha(x)T_{\beta\gamma}(y)T_{\rho\sigma}(z) \rangle$ by expanding (14) around flat space to second order. Using Ward identities for the conservation of $T_{\mu\nu}$, one can then derive a relation on two point functions which can only be satisfied if the coefficients c in (17) and (14) are identical.

¹We thank D. Anselmi for pointing out the relevance of the Adler-Bardeen theorem.

²The field normalization conventions in [43] differ from those used here.

We are finally ready to compute the cross-section. Fourier transforming the two-point function (17), one obtains

$$\Pi_{\alpha\beta\gamma\delta}(p) = \int d^4x e^{ip\cdot x} \langle T_{\alpha\beta}(x) T_{\gamma\delta}(0) \rangle = \frac{c}{48\pi^4} \hat{X}_{\alpha\beta\gamma\delta} \int d^4x \frac{e^{ip\cdot x}}{x^4} \quad (19)$$

where $\hat{X}_{\alpha\beta\gamma\delta}$ is just the $X_{\alpha\beta\gamma\delta}$ of (18) with $\partial \rightarrow -ip$. The integral is evaluated formally as

$$\Pi(s) = \int d^4x \frac{e^{ip\cdot x}}{x^4} = \pi^2 \log(-s) + (\text{analytic in } s) \quad (20)$$

where, as before, $s = p^2$. One easily reads off the discontinuity across the positive real axis in the s -plane:

$$\text{Disc } \Pi(s) = \Pi(s + i\varepsilon) - \Pi(s - i\varepsilon) = -2\pi^3 i, \quad (21)$$

and so

$$\text{Disc } \Pi_{\alpha\beta\gamma\delta}(p) = -\frac{ic}{24\pi} \hat{X}_{\alpha\beta\gamma\delta}. \quad (22)$$

For the sake of definiteness, let us consider a graviton polarized in the x^1 - x^2 direction. $\hat{X}_{1212} = -3\omega^4$ for normal incidence, so

$$\sigma = \frac{2\kappa^2}{2i\omega} \text{Disc } \Pi_{1212}(p) \Big|_{\substack{p^0=\omega \\ \vec{p}=0}} = \frac{c}{8\pi} \kappa^2 \omega^3, \quad (23)$$

in agreement with the classical result when $c = N^2/4$. The extra factor of $2\kappa^2$ in the second expression in (23) comes from the fact that $h_{\alpha\beta}$ as defined in the text following (11) is not a canonically normalized scalar field; instead, $h_{\alpha\beta}/\sqrt{2\kappa^2}$ is.

In summary, the non-renormalization argument is as follows: the graviton cross-section is read off at leading order in κ , but correct to all orders in g_{str} , from the two-point function of the stress-energy tensor. The two-point function is not renormalized beyond one loop because the Schwinger term in the OPE (16) is similarly non-renormalized. That in turn is due to the fact that the central charge appearing in the Schwinger term is precisely the coefficient of the Weyl tensor squared in the trace anomaly (14). The trace anomaly is not renormalized past one loop because T_α^α is related by supersymmetry to the divergence of the $SU(4)$ R -current, which is protected by the Adler-Bardeen theorem against anomalies beyond one loop. Another, more heuristic, reason why the central charge should not be renormalized is that there is a critical line extending from $g_{\text{str}}N = 0$ to $g_{\text{str}}N = \infty$. The central charge is expected to be constant along a critical line. In four dimensions, this expectation is supported by the work of [43].³ Therefore, c can be calculated in the $g_{\text{str}}N \rightarrow 0$ limit where it is given by one-loop diagrams.

We could adopt a different strategy and invert our arguments. Requiring that the world volume theory of N coincident 3-branes agrees with semiclassical supergravity tells us that in

³A more definitive argument has been given in four dimensions for the constancy of flavor central charges along fixed lines [44].

the $g_{\text{str}}N \rightarrow \infty$ limit its central charge approaches $N^2/4$. In the next section we will similarly deduce the Schwinger terms in the two-point functions of the stress-energy tensor for other branes. Furthermore, we can study two-point functions of other operators by calculating the semiclassical absorption cross-section for particles that couple to them. For instance, the dilaton couples to $\text{tr } F^2$. The dilaton absorption cross-section was calculated in [29]. The semiclassical result implies that the Schwinger term here is again $\sim N^2$ in the $g_{\text{str}}N \rightarrow \infty$ limit. Comparison with the gauge theory calculation [29, 30] suggests that the one-loop result is again exact. It will be interesting to extract more results from these connections between gravity and gauge theory.

3 Extension to Other Branes

In this section we further explore the connection between the absorption cross-sections in semiclassical supergravity and central terms in two-point functions calculated in corresponding world volume theories. We proceed in analogy to the 3-brane discussion presented in the previous section, and consider absorption of gravitons polarized along the branes. In the world volume theories such gravitons couple to components of stress-energy tensor, $T_{\alpha\beta}$. On the other hand, in supergravity such gravitons satisfy the minimally coupled scalar equation with respect to the coordinates transverse to the brane [30]. This establishes a general connection between the absorption cross-section of a minimally coupled scalar and the central term in the algebra of stress-energy tensors. For N coincident 3-branes the world volume theory is known, and we have shown that the conformal anomaly is in exact agreement with this principle. There are cases, however, where little is known about the world volume theory of multiple branes. In such cases we can use our method to find the Schwinger term without knowing any details of the world volume theory.

Consider, for instance, the 2-branes and the 5-branes of M-theory. While the world volume theory of multiple coincident branes is not known in detail, the extreme supergravity solutions are well-known. The absorption cross-sections for low-energy gravitons polarized along the brane were calculated in [29, 30, 36], with the results

$$\sigma_2 = \frac{1}{6\sqrt{2}\pi} \kappa_{11}^2 \omega^2 N^{3/2} , \quad \sigma_5 = \frac{1}{3 \cdot 2^6 \pi^2} N^3 \omega^5 . \quad (24)$$

For N coincident M2-branes we may deduce that the schematic structure of the stress-energy tensor OPE is

$$T(x)T(0) = \frac{c_2}{x^6} + \dots \quad (25)$$

where the central charge behaves as $c_2 \sim N^{3/2}$ in the large N limit. For N coincident M5-branes we instead have

$$T(x)T(0) = \frac{c_5}{x^{12}} + \dots \quad (26)$$

Now the central charge behaves as $c_5 \sim N^3$ in the large N limit. These results have an obvious connection with properties of the near-extremal entropy found in [28]. Indeed, the

near-extremal entropy of a large number N of coincident M2-branes is formally reproduced by $\mathcal{O}(N^{3/2})$ massless free fields in 2+1 dimensions, while that of N coincident M5-branes is reproduced by $\mathcal{O}(N^3)$ massless free fields in 5+1 dimensions.

As a final example we consider the 5-branes of string theory. In [46] it was shown that their near-extremal entropy is reproduced by a novel kind of string theory, rather than by massless fields in 5+1 dimensions. For N coincident D5-branes the string tension turns out to be that of a D-string divided by N . This suggests that the degrees of freedom responsible for the near-extremal entropy are those of “fractionated” D-strings bound to the D5-branes. An S-dual of this picture suggests that the entropy of multiple NS-NS 5-branes comes from fractionated fundamental strings bound to them.⁴ We would like to learn more about these theories by probing them with longitudinally polarized gravitons incident transversely to the brane.

The extreme Einstein metric of both the NS-NS and the R-R 5-branes is

$$ds_E^2 = \left(1 + \frac{R^2}{r^2}\right)^{-1/4} (-dt^2 + dx_1^2 + \dots + dx_5^2) + \left(1 + \frac{R^2}{r^2}\right)^{3/4} (dr^2 + r^2 d\Omega_3^2) . \quad (27)$$

The s-wave Laplace equation in the background of this metric is

$$\left[\rho^{-3} \frac{d}{d\rho} \rho^3 \frac{d}{d\rho} + 1 + \frac{(\omega R)^2}{\rho^2} \right] \phi(\rho) = 0 , \quad (28)$$

where $\rho = \omega r$. Remarkably, this equation is exactly solvable in terms of Bessel functions. The two possible solutions are

$$\rho^{-1} J_{\pm \sqrt{1 - (\omega R)^2}}(\rho) . \quad (29)$$

Clearly, there are two physically different regimes. For $\omega R > 1$ the label of the Bessel function is imaginary, and the requirement that the wave is incoming for $\rho \rightarrow 0$ selects the solution

$$\rho^{-1} J_{-i\sqrt{(\omega R)^2 - 1}}(\rho) . \quad (30)$$

From the large ρ asymptotics we find that the absorption probability is

$$\mathcal{P} = 1 - e^{-2\pi\sqrt{(\omega R)^2 - 1}} . \quad (31)$$

Hence, for $\omega R \geq 1$ the absorption cross-section is

$$\sigma = \frac{4\pi}{\omega^3} \left(1 - e^{-2\pi\sqrt{(\omega R)^2 - 1}} \right) . \quad (32)$$

For $\omega R < 1$ the question of how to choose the solution is somewhat more subtle. It is clear that $\rho^{-1} J_{\sqrt{1 - (\omega R)^2}}(\rho)$ is better behaved near $\rho = 0$ than $\rho^{-1} J_{-\sqrt{1 - (\omega R)^2}}(\rho)$. If we approach the extreme 5-brane as a limit of a near-extreme 5-brane, we indeed find that $\rho^{-1} J_{\sqrt{1 - (\omega R)^2}}(\rho)$

⁴New insights into the world volume theory of NS-NS 5-branes were recently obtained in [47, 48, 49].

is the solution that is selected. Since this solution is real, there is no absorption for $\omega R < 1$.⁵ This result agrees with the extremal limit of the 5-brane absorption cross-section calculated in [17].

It is not hard to generalize our calculation to higher partial waves. For the ℓ -th partial wave we find that the absorption cross-section vanishes for $\omega R \leq \ell + 1$. Above this threshold it is given by

$$\sigma_\ell = \frac{4\pi(\ell+1)^2}{\omega^3} \left(1 - e^{-2\pi\sqrt{(\omega R)^2 - (\ell+1)^2}} \right). \quad (33)$$

From (32) we reach the surprising conclusion that

$$\langle T(\omega, \vec{0}) T(-\omega, \vec{0}) \rangle \quad (34)$$

vanishes identically for $\omega < 1/R$, which implies that gravity does not couple to the massless modes of the world volume theory! The threshold energy $1/R$ is precisely $1/\sqrt{\alpha'_{\text{eff}}}$, where $\alpha'_{\text{eff}} = 1/(2\pi T_{\text{eff}})$ and

$$T_{\text{eff}} = \frac{1}{2\pi R^2} \quad (35)$$

is the tension of the fractionated strings. The ordinary superstring has its first massive excited state at mass $m^2 = 2/\alpha'$. The threshold energy squared is half this value with α' replaced by α'_{eff} . If one imagines producing a single massive string at $\omega = 1/R$, then its mass is $m = 1/\sqrt{\alpha'_{\text{eff}}}$. Perhaps this is the first excited level of the non-critical string living on the 5-brane. Similarly, the higher partial wave thresholds might correspond to higher excited levels of mass $(\ell+1)/\sqrt{\alpha'_{\text{eff}}}$. If instead $\omega \geq 1/R$ corresponds to the pair production threshold of the first massive state of fractionated strings, then the mass would be $m = 1/(2\sqrt{\alpha'_{\text{eff}}})$. Neither picture yields any obvious explanation of the behavior

$$\langle T(\omega, \vec{0}) T(-\omega, \vec{0}) \rangle \sim [(\omega R)^2 - 1]^{1/2} \quad (36)$$

just above threshold. Pair production in a weakly interacting theory would predict a $7/2$ power in (36). It would be interesting to find an explanation of the observed square-root scaling in (36).

We believe that our discussion applies to a large number of coincident D5-branes, as well as to solitonic 5-branes of type IIA and IIB theories. This is because all these solutions have the same Einstein metric. Perhaps in the large N limit some properties of the world volume theories of these different branes become identical.⁶ For N coincident NS-NS 5-branes,

$$R^2 \sim N\alpha' . \quad (37)$$

Thus, the absorption cross-section (32) is formally independent of g_{str} . Therefore, our formula should be applicable in the $g_{\text{str}} \rightarrow 0$ limit proposed in [48]. There it was argued that in this

⁵The fact that an extremal 5-brane in 10 dimensions does not absorb minimally coupled scalars below a certain threshold was noted in [50].

⁶We thank J. Maldacena for suggesting this possibility to us.

limit the NS-NS 5-branes decouple from the bulk modes. We indeed find that incident gravitons are not absorbed for sufficiently low energies. However, above a critical energy of order $1/\sqrt{N\alpha'}$ the 5-branes do appear to couple to the bulk modes. As we have commented, this is probably related to the fact that the scale of the string theory living on the 5-brane is

$$\alpha'_{\text{eff}} = N\alpha' , \quad (38)$$

i.e. the fundamental strings become fractionated [46].

In summary, we note that probing branes with low-energy particles incident from transverse directions is a useful tool for extracting correlation functions in their world volume theory. Here we have given an application of this technique to M2-branes and to 5-branes, but a more general investigation would be worthwhile.

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References

- [1] M. Cvetič and A.A. Tseytlin, Phys. Lett. **B366** (1996) 95, hep-th/9510097; Phys. Rev. **D53** (1996) 5619, hep-th/9512031.
- [2] A. Strominger and C. Vafa, Phys. Lett. **B379** (1996) 99, hep-th/9601029.
- [3] C.G. Callan and J.M. Maldacena, Nucl. Phys. **B472** (1996) 591, hep-th/9602043.
- [4] J.M. Maldacena and A. Strominger, Phys. Rev. Lett. **77** (1996) 428, hep-th/9603060.
- [5] C. Johnson, R. Khuri and R. Myers, Phys. Lett. **B378** (1996) 78, hep-th/9603061.
- [6] A.A. Tseytlin, Nucl. Phys. **B475** (1996) 49, hep-th/9604035.
- [7] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B475** (1996) 179, hep-th/9604166.
- [8] V. Balasubramanian and F. Larsen, Nucl. Phys. **B478** (1996) 199, hep-th/9604189.
- [9] M. Cvetič and D. Youm, Phys. Rev. **D54** (1996) 2612, hep-th/9603147.
- [10] A. Dhar, G. Mandal and S. R. Wadia, Phys. Lett. **B388** (1996) 51, hep-th/9605234.

- [11] S.R. Das and S.D. Mathur, Nucl. Phys. **B478** (1996) 561, hep-th/9606185.
- [12] S.S. Gubser and I.R. Klebanov, Nucl. Phys. **B482** (1996) 173, hep-th/9608108.
- [13] J.M. Maldacena and A. Strominger, Phys. Rev. **D55** (1997) 861, hep-th/9609026.
- [14] J M. Maldacena and A. Strominger, hep-th/9702015.
- [15] S.S. Gubser and I.R. Klebanov, Phys. Rev. Lett. **77** (1996) 4491, hep-th/9609076.
- [16] C.G. Callan, Jr., S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B489** (1997) 65, hep-th/9610172; I.R. Klebanov and M. Krasnitz, Phys. Rev. **D55** (1997) 3250, hep-th/9612051.
- [17] I.R. Klebanov and S. Mathur, Nucl. Phys. **B500** (1997) 115, hep-th/9701187.
- [18] S. Hawking and M. Taylor-Robinson, hep-th/9702045.
- [19] F. Dowker, D. Kastor and J. Traschen, hep-th/9702109.
- [20] M. Krasnitz and I.R. Klebanov, Phys. Rev. **D56** (1997) 2173, hep-th/9703216.
- [21] I.R. Klebanov, A. Rajaraman and A. Tseytlin, hep-th/9704112.
- [22] S. Mathur, hep-th/9704156.
- [23] S. S. Gubser, hep-th/9704195.
- [24] S. S. Gubser, hep-th/9706100.
- [25] M. Cvetič and F. Larsen, hep-th/9705192, hep-th/9706071.
- [26] S. Ferrara and R. Kallosh, hep-th/9602136, hep-th/9603090; B. Kol and A. Rajaraman, hep-th/9608126.
- [27] S.S. Gubser, I.R. Klebanov and A.W. Peet, Phys. Rev. **D54** (1996) 3915, hep-th/9602135.
- [28] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B475** (1996) 165, hep-th/9604089.
- [29] I.R. Klebanov, Nucl. Phys. **496** (1997) 231, hep-th/9702076.
- [30] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B499** (1997) 217, hep-th/9703040.
- [31] J. Dai, R. G. Leigh, and J. Polchinski, Mod. Phys. Lett. **A4**, 2073 (1989);
R. G. Leigh, Mod. Phys. Lett. **A4**, 2767 (1989).
- [32] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).

- [33] E. Witten, Nucl. Phys. **B460** (1996) 541.
- [34] G.W. Gibbons, G.T. Horowitz and P.K. Townsend, hep-th/9410073.
- [35] S. Das, hep-th/9703146.
- [36] R. Emparan, hep-th/9706204.
- [37] A.A. Tseytlin, Nucl. Phys. **B469** (1996) 51, hep-th/9602064.
- [38] C. G. Callan, private communication.
- [39] C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (NY) **59** (1970) 42.
- [40] M. F. Sohnius, Phys. Rep. 128 (1985) 39.
- [41] D. Anselmi, M. T. Grisaru, and A. A. Johansen, hep-th/9601023.
- [42] J. Erdmenger and H. Osborn, Nucl. Phys. **B483** (1997) 431, hep-th/9605009.
- [43] D. Anselmi, D. Z. Freedman, M. T. Grisaru, and A. A. Johansen, hep-th/9608125.
- [44] D. Anselmi, D. Z. Freedman, M. T. Grisaru, and A. A. Johansen, hep-th/9708042.
- [45] P. C. W. Birrell and N. D. Davies, *Quantum Fields in Curved Space* (Cambridge, UK: Cambridge University Press, 1982).
- [46] J. Maldacena, Nucl. Phys. **B477** (1996) 168, hep-th/9605016.
- [47] R. Dijkgraaf, E. Verlinde and H. Verlinde, hep-th/9704018.
- [48] N. Seiberg, hep-th/9705221; O. Aharony, M. Berkooz, S. Kachru, N. Seiberg, E. Silverstein, hep-th/9707079.
- [49] E. Witten, hep-th/9707093.
- [50] V. Balasubramanian and F. Larsen, Nucl. Phys. **B495** (1997) 206 hep-th/9610077.